

MARKER CODE				



STUDENT ENROLMENT NUMBER (SEN)									

TONGA NATIONAL FORM SEVEN CERTIFICATE

MATHEMATICS with CALCULUS

2017

QUESTION and ANSWER BOOKLET

Time allowed: 3 Hours

INSTRUCTIONS:

1. Write your **Student Enrolment Number (SEN)** on the top right hand corner of this booklet.
2. ANSWER ALL QUESTIONS. Write your answers for each question in the spaces provided in this booklet.
3. If you need more space for answers, ask the Supervisor for extra paper. Write your SEN on all extra sheets used and clearly number the questions. Attach the extra sheets at the appropriate places in this booklet.
4. Show all working. Unless otherwise stated, numerical answers correct to three significant figures will be adequate.

TOPICS	Total
QUESTION ONE : NUMBERS	9
QUESTION TWO : ALGEBRA	18
QUESTION THREE : TRIGONOMETRY	16
QUESTION FOUR : DIFFERENTIATION	13
QUESTION FIVE : INTEGRATION	24
TOTAL	80

Check that this booklet contains pages 2-23 in the correct order and that pages 22-23 has been deliberately left blank.

A 4-page booklet (No. 108/2) containing mathematical formulae and tables has been provided.

YOU MUST HAND IN THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE: NUMBERS

1. Write $\frac{2}{3}\log 8 - \frac{4}{5}\log 32 + \frac{1}{7}\log 128$ in the form $\log a$.

Skill level 2	
2	
1	
0	
NR	

2. If $w=3+5i$ and $z=2+3i$ evaluate $\frac{w}{z}$. Write your answer in the form $a+ib$.

Skill level 2	
2	
1	
0	
NR	

3. If $z=x+iy$ and $\bar{z}$ is the conjugate of z , find $3z(2-\bar{z})$ in terms of x and y .

Skill level 2	
2	
1	
0	
NR	

4. Evaluate $(3 - i\sqrt{3})^8$. Write your answer in the form $a + ib$.

Skill level 3	
3	
2	
1	
0	
NR	

Skill level 2	
2	
1	
0	
NR	

Skill level 2	
2	
1	
0	
NR	

3. When $x^3 - 3mx^2 - 2nx + 3$ is divided by $x + 1$ it has a remainder of 4, and when $x^3 - 2x^2 - 2nx + m$ is divided by $x - 2$ the remainder is -3 .

Find m and n .

This image shows a blank sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There are approximately 20 lines visible. The paper has a slight shadow on the right side, suggesting it's resting on a surface.

Skill level 3	
3	
2	
1	
0	
NR	

- 4.

Skill level 3	
3	
2	
1	
0	
NR	

- 5.

Skill level 3	
3	
2	
1	
0	
NR	

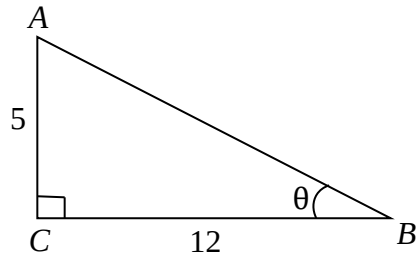
6. Prove by induction that $\sum_{r=1}^n 2r(3r-1) = 4+20+40+\dots+2n(3n-1) = 2n^2(n+1)$ for any positive integer n .

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

	Skill level 4	
	4	
	3	
	2	
	1	
	0	
	NR	

QUESTION THREE: TRIGONOMETRY

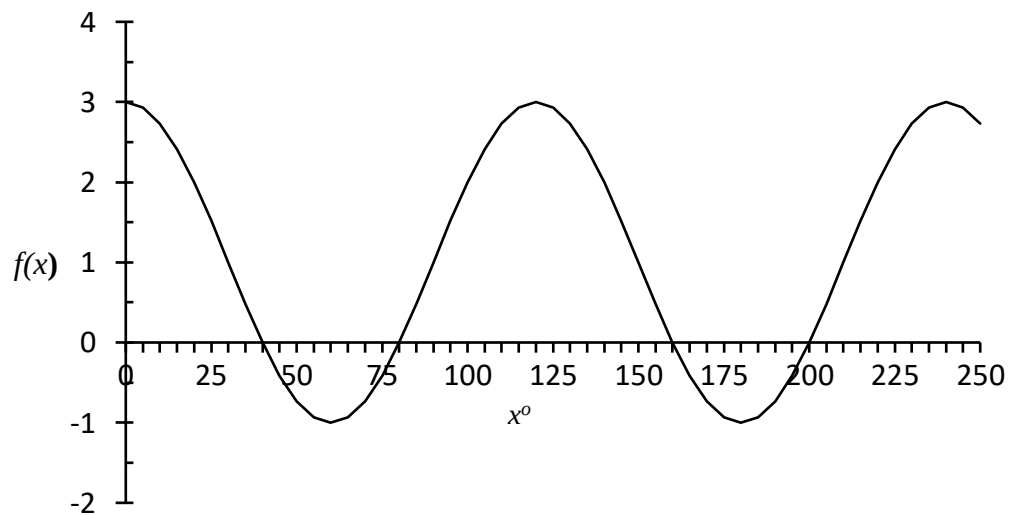
1. The right angled triangle ABC shown has two known sides, $\overline{AC}=5$ and $\overline{BC}=12$



Evaluate $\sin \theta + 2 \cos \theta - 3 \tan \theta$. Leave your answer as a fraction in its simplest form.

Skill level 2	
2	
1	
0	
NR	

2. The graph drawn is the function $f(x) = 2 \cos(3x) + 1$ where $0^\circ \leq x \leq 250^\circ$.



- i. What is the period of $f(x)$?

Skill level 1	
1	
0	
NR	

Skill level 3	
3	
2	
1	
0	
NR	

3. Solve $2 \cos^2 x - \cos x - 1 = 0$, $0 \leq x \leq 2\pi$.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Skill level 2	
2	
1	
0	
NR	

4. Solve $\sin 2\theta - \cos \theta = 0$, $-\pi \leq \theta \leq \pi$.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Skill level 2	
2	
1	
0	
NR	

5. Use an appropriate identity to express $2\sin\frac{\pi}{3}\cos\frac{\pi}{4}$ as a sum of two angles.

[illegible]

Skill level 2	
2	
1	
0	
NR	

6. Prove the trigonometric identity: $\frac{1 - \sin^2 \theta + \cos^2 \theta}{\sin 2\theta} - \tan \theta = 2 \cot(2\theta)$.

[illegible]

Skill level 3	
3	
2	
1	
0	
NR	

QUESTION FOUR: DIFFERENTIATION

1. Differentiate $3x^3 - x^{-1}$.

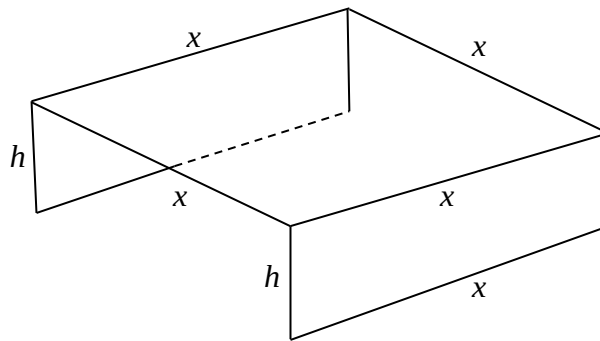
Skill level 1	
1	
0	
NR	

2. Evaluate $\lim_{x \rightarrow -3} \left(\frac{x+3}{x^2-9} \right)$.

Skill level 2	
2	
1	
0	
NR	

3. Find the equation of the normal line to the curve $x^2 - 2y^3 = -7$ at $(3,2)$.

Skill level 3	
3	
2	
1	
0	
NR	

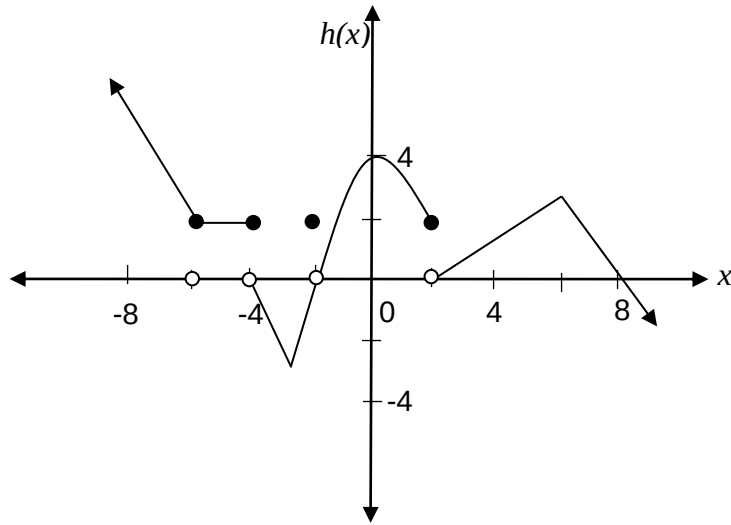


The tent is to be constructed by 900 m² of materials. Find the maximum volume of the tent that could be constructed by the materials.

[illegible]

Skill level 4	
4	
3	
2	
1	
0	
NR	

5. The graph of the function $h(x)$ is drawn below. Use the graph to answer the following questions.



- i. Give two values of x in which $h(x)$ is continuous but not differentiable?

Skill level 1	
1	
0	
NR	

- ii. Evaluate $\lim_{x \rightarrow -2} h(x)$.

Skill level 1	
1	
0	
NR	

- iii. Solve $h(x) = 0$.

Skill level 1	
1	
0	
NR	

QUESTION FIVE: INTEGRATION

1. Find the following indefinite integrals.

i. $\int (x^3 + 4 e^{2x}) dx$

Skill level 2	
2	
1	
0	
NR	

ii. $\int \cos^3 x dx$

Skill level 3	
3	
2	
1	
0	
NR	

2. Given $\int 2x^3 \sqrt{x^2 - 1} \, dx$. Do the following and thus find an answer for the given indefinite integral.

- i. Rewrite the given indefinite integral in terms of variable u only by letting $u = x^2 - 1$.

[illegible]

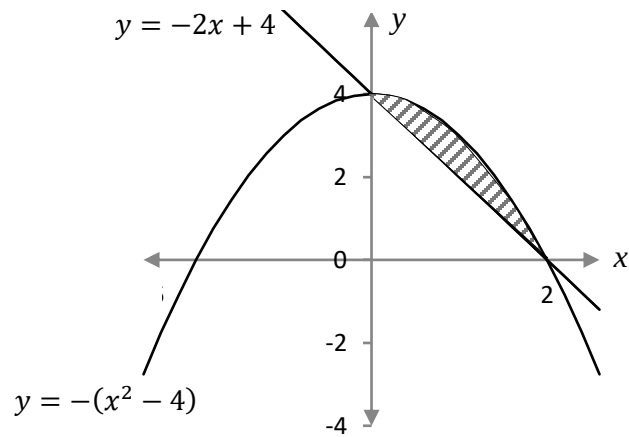
Skill level 1	
1	
0	
NR	

- ii. Integrate the indefinite integral you have obtained in part i. and thus find an answer for the given indefinite integral.

[illegible]

Skill level 2	
2	
1	
0	
NR	

3. The area enclosed by the graphs of $y = -(x^2 - 4)$ and $y = -2x + 4$ is shaded on the given diagram.



- i. Calculate the area enclosed by the graphs $y = -(x^2 - 4)$ and $y = -2x + 4$.

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Skill level 3	
3	
2	
1	
0	
NR	

- ii. Find the volume of the solid formed when the shaded area is rotated about the y -axis.

[illegible]

Skill level 4	
4	
3	
2	
1	
0	
NR	

4. Find $\int \frac{x+11}{2x^2-5x-3} dx$. [Hint: Apply the partial fraction method]

[illegible]

Skill level 4	
4	
3	
2	
1	
0	
NR	

5. The table below gives the values of $x^2 \sin x$ from 1 and 2 in steps of 0.2.

x	1.0	1.2	1.4	1.6	1.8	2.0
$x^2 \sin x$	0.84	1.34	1.93	2.56	3.16	3.64

Note: x is in radians

Approximate $\int_1^2 x^2 \sin(x) dx$ using the Simpson rule.

[illegible]

Skill level 2	
2	
1	
0	
NR	

6. Solve $x \frac{dy}{dx} + y = 2xy$ and find a particular solution given $y = 1$ when $x = 1$.

[illegible]

Skill level 3	
3	
2	
1	
0	
NR	

THIS PAGE HAS BEEN DELIBERATELY LEFT BLANK.

THIS PAGE HAS BEEN DELIBERATELY LEFT BLANK.